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Sterile neutrino creating a reduced LSND effect*

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Abstract

Although the hypothetic sterile neutrino ν_s is probably not involved significantly in the deficits of solar ν_e's and atmospheric ν_μ's, it may cause the possible LSND effect. In fact, we face such a situation, when the popular nearly bimaximal texture of active neutrinos ν_e , ν_μ , ν_τ is perturbed through a small rotation in the 14 plane, where ν_4 is the extra neutrino mass state induced by the sterile neutrino ν_s. Then, with $m_1^2 \simeq m_2^2$ we predict in the simplest case of $s_{13} \to 0$ that $\sin^2 2\theta_{\rm LSND} = s_{14}^4/2$ and $\Delta m^2_{\rm LSND} = |\Delta m^2_{41}|$. However, the negative Chooz experiment imposes on $s_{14}^4/2$ the upper bound 1.3×10^{-3}, suggesting a reduction of the amplitude of possible LSND effect.

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As is well known, the existence of at least one sterile neutrino ν_s is needed — beside three familiar active neutrinos $\nu_e\,,\,\nu_\mu\,,\,\nu_\tau$ — in order to explain in terms of neutrino oscillations not only the observed deficits of solar ν_e's and atmospheric ν_μ's, but also the possible LSND effect for accelerator $\bar{\nu}_\mu$'s and ν_μ's. In this note we consider the phenomenologically popular nearly bimaximal texture of three active neutrinos [1] describing nicely both deficits,

$$U^{(0)} = \left(\begin{array}{cccc} c_{13}/\sqrt{2} & c_{13}/\sqrt{2} & s_{13} & 0 \\ -(1+s_{13})/2 & (1-s_{13})/2 & c_{13}/\sqrt{2} & 0 \\ (1-s_{13})/2 & -(1+s_{13})/2 & c_{13}/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \,, \tag{1}$$

where $c_{12} = 1/\sqrt{2} = s_{12}$ and $c_{23} = 1/\sqrt{2} = s_{23}$ with $\delta_{13} = 0$, and then we perturb it through multiplying on the right by the rotation in the 14 plane with $\delta_{14} = 0$. This results into the following four-neutrino texture:

$$U = \left(\begin{array}{cccc} c_{13}c_{14}/\sqrt{2} & c_{13}/\sqrt{2} & s_{13} & c_{13}s_{14}/\sqrt{2} \\ -(1+s_{13})c_{14}/2 & (1-s_{13})/2 & c_{13}/\sqrt{2} & -(1+s_{13})s_{14}/2 \\ (1-s_{13})c_{14}/2 & -(1+s_{13})/2 & c_{13}/\sqrt{2} & (1-s_{13})s_{14}/2 \\ -s_{14} & 0 & 0 & c_{14} \end{array} \right) \,. \tag{2}$$

Here, of course, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Such a four-neutrino mixing matrix can be obtained from its generic form by additionally putting $s_{24} = 0$ and $s_{34} = 0$. With $U = (U_{\alpha i})$ we can write

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i \,, \tag{3}$$

where $\nu_\alpha = \nu_e\,,\,\nu_\mu\,,\,\nu_\tau\,,\,\nu_s$ and $\nu_i = \nu_1\,,\,\nu_2\,,\,\nu_3\,,\,\nu_4$ are the flavor and mass neutrinos, respectively.

In the following we will discuss the consequences of the texture (2), showing that both s_{13} and s_{14} should be small to describe reasonably neutrino data (s_{13} might even vanish). In particular, the LSND effect, if confirmed, becomes a "sterile" perturbation of the nearly bimaximal texture (1) with the amplitude $\sin^2 2\theta_{\rm LSND} = (1+2s_{13})s^4_{14}/2$ and mass-square scale $\Delta m^2_{\rm LSND} = |m^2_4 - m^2_1|$, while the solar and atmospheric mass-square scales are $\Delta m^2_{\rm sol} = m^2_2 - m^2_1$ and $\Delta m^2_{\rm atm} = m^2_3 - m^2_2$, respectively, where $m^2_1 \stackrel{<}{\sim} m^2_2 \ll m^2_3$. Thus, the sterile neutrino ν_s is here needed only to create the possible LSND effect.

We start from the familiar formulae for probabilities of neutrino oscillations $\nu_\alpha \rightarrow \nu_\beta$ on the energy shell:

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | e^{iPL} | \nu_\alpha \rangle|^2 = \delta_{\beta\alpha} - 4 \sum_{j>i} U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i} \sin^2 x_{ji} \; , \tag{4}$$

valid if the quartic product $U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i}$ is real, what is certainly true when a possible CP violation can be ignored [then $U^*_{\alpha i} = U_{\alpha i}$, as in the case of Eq. (2), and $P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$]. Here,

$$x_{ji} = 1.27 \frac{\Delta m^2_{ji} L}{E} \;\; , \;\; \Delta m^2_{ji} = m^2_j - m^2_i \tag{5}$$

with Δm^2_{ji}, L and E measured in eV^2, km and GeV, respectively (L and E denote the experimental baseline and neutrino energy, while $p_i = \sqrt{E^2 - m^2_i} \simeq E - m^2_i/2E$ are eigenstates of the neutrino momentum P).

With the use of oscillation formulae (4), the proposal (2) for the four-neutrino mixing matrix leads, in particular, to the following probabilities if $m^2_1 \simeq m^2_2$:

$$\begin{aligned}
P(\nu_e \;\rightarrow\; \nu_e) \;&\simeq\; 1 - c^4_{13} c^2_{14} \sin^2 x_{21} - c^2_{13} \left(1 + c^2_{14}\right) \left(2 s^2_{13} \sin^2 x_{32} + c^2_{13} s^2_{14} \sin^2 x_{41}\right) \\
&\quad - 2 c^2_{13} s^2_{13} s^2_{14} \sin^2 x_{43} \; , \\
P(\nu_{\mu,\tau} \rightarrow \nu_{\mu,\tau}) \;&\simeq\; 1 - \frac{1}{4}\, c^4_{13} c^2_{14} \sin^2 x_{21} \\
&\quad - \left[\frac{1}{2}(1 + s^2_{13})(1 + c^2_{14}) \mp s_{13} s^2_{14}\right] \left[c^2_{13} \sin^2 x_{32} + \frac{1}{2}(1 \pm s_{13})^2 s^2_{14} \sin^2 x_{41}\right] \\
&\quad - \frac{1}{2} c^2_{13} (1 \pm s_{13})^2 s^2_{14} \sin^2 x_{43} \; , \\
P(\nu_{\mu,\tau} \rightarrow \;\nu_e) \;&\simeq\; \frac{1}{2}\, c^4_{13} c^2_{14} \sin^2 x_{21} \\
&\quad + c^2_{13} \left[s^2_{14} \mp s_{13}(1 + c^2_{14})\right] \left[\mp s_{13} \sin^2 x_{32} + \frac{1}{2}(1 \pm s_{13}) s^2_{14} \sin^2 x_{41}\right] \\
&\quad \pm c^2_{13} s_{13} (1 \pm s_{13}) s^2_{14} \sin^2 x_{43} \, .
\end{aligned} \tag{6}$$

Note also that $P(\nu_e \rightarrow \nu_s) = 2 c^2_{13} c^2_{14} s^2_{14} \sin^2 x_{41}$. Here, $P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$ as a possible PC violation is neglected.

For solar experiments [2], under the assumption that $(x_{21})_{\rm sol} \sim 1 \ll (x_{32})_{\rm sol}$, $|(x_{41})_{\rm sol}|$, $|(x_{43})_{\rm sol}|$, the first Eq. (6) gives

$$
\begin{aligned}
P(\nu_e \to \nu_e)_{\rm sol} &\simeq 1 - c_{13}^4 c_{14}^2 \sin^2(x_{21})_{\rm sol} - 2(c_{13}s_{13})^2 - \frac{1}{2}c_{13}^4(1+c_{14}^2)s_{14}^2 \\
&\simeq 1 - c_{14}^2 \sin^2(x_{21})_{\rm sol} - \frac{1}{2}(1+c_{14}^2)s_{14}^2 \,, \qquad (7)
\end{aligned}
$$

where the second step is valid for $c_{13}^2 \gg s_{13}^2 \simeq 0$. Treating Eq. (7) perturbatively with respect to the constant term proportional to s_{14}^2 (and neglecting s_{13}^2), we get in the zero perturbative order

$$
c_{14}^2 = \sin^2 2\theta_{\rm sol} \sim 0.66 \text{ or } 0.97 \text{ or } 0.80 \;,\; \Delta m_{21}^2 = \Delta m_{\rm sol}^2 \sim (10^{-5} \text{ or } 10^{-7} \text{ or } 10^{-10})\ \mathrm{eV}^2 \qquad (8)
$$

and so,

$$
\frac{1}{2}(1+c_{14}^2)s_{14}^2 \sim 0.28 \;\text{ or }\; 0.030 \;\text{ or }\; 0.18 \;, \qquad (9)
$$

if the recent estimation [3] of two-flavor LMA or LOW or VAC solar solution is used, respectively. Thus, this treatment is reasonable for LOW solar solution. In contrast, for LMA and VAC solar solutions the constant term in Eq. (7), modifying the familiar two-flavor structure of oscillation formula, cannot be neglected and so, in our four-neutrino texture these solar solutions cannot be applied without some changes tending to introduce into solar solutions a third additive parameter.

In the case of atmospheric experiments [2], under the assumption that $(x_{21})_{\rm atm} \ll (x_{32})_{\rm atm} \sim 1 \ll |(x_{41})_{\rm atm}|$, $|(x_{43})_{\rm atm}|$, the second Eq. (6) implies

$$
\begin{aligned}
P(\nu_\mu \to \nu_\mu)_{\rm atm} &\simeq 1 - c_{13}^2\left[\frac{1}{2}(1+s_{13}^2)(1+c_{14}^2) - s_{13}s_{14}^2\right]\sin^2(x_{32})_{\rm atm} \\
&\quad -\frac{1}{2}\left[1 - \frac{1}{4}(1+s_{13})^2 s_{14}^2\right](1+s_{13})^2 s_{14}^2 \\
&\simeq 1 - \left[\frac{1}{2}(1+c_{14}^2) - s_{13}s_{14}^2\right]\sin^2(x_{32})_{\rm atm} \\
&\quad - \left[\frac{1}{8}(3+c_{14}^2) + \frac{1}{2}s_{13}(1+c_{14}^2)\right] s_{14}^2 \,, \qquad (10)
\end{aligned}
$$

the second step being valid for $c_{13}^2 \gg s_{13}^2 \simeq 0$ in the linear approximation in s_{13}. Then, in the zero perturbative order in the constant term proportional to s_{14}^2 (and neglecting s_{13}^2, but not s_{13}) we obtain

$$\sin^2 2\theta_{\rm atm} = \frac{1}{2}(1+c^2_{14}) - s_{13}s^2_{14} \sim 0.99 - 0.030 s_{13}\;,\;\Delta m^2_{32} = \Delta m^2_{\rm atm} \sim 3\times 10^{-3}\;{\rm eV}^2 \quad (11)$$

and so,

$$\frac{1}{8}(3+c^2_{14})s^2_{14} + \frac{1}{2}s_{13}(1+c^2_{14})s^2_{14} \sim 0.015 + 0.030 s_{13}\;, \quad (12)$$

if the LOW solar solution is used for c^2_{14}.

Eventually, for the LSND accelerator experiment [4], under the assumption that $(x_{21})_{\rm LSND} \ll (x_{32})_{\rm LSND} \ll |(x_{43})_{\rm LSND}| \simeq |(x_{41})_{\rm LSND}| \sim 1$ the third Eq. (6) leads to

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e)_{\rm LSND} &\simeq \frac{1}{2}c^2_{13}(1+s_{13})^2 s^4_{14}\sin^2(x_{41})_{\rm LSND} \\ &\simeq \frac{1}{2}(1+2s_{13})s^4_{14}\sin^2(x_{41})_{\rm LSND}\;, \end{aligned} \quad (13)$$

where the second step holds for $c^2_{13} \gg s^2_{13} \simeq 0$ in the linear approximation in s_{13}. Hence, the prediction

$$\sin^2 2\theta_{\rm LSND} = \frac{1}{2}\,c^2_{13}(1+s_{13})^2 s^4_{14} \simeq \frac{1}{2}(1+2s_{13})s^4_{14} \sim 4.5(1+2s_{13})\times 10^{-4} \quad (14)$$

follows for the amplitude of LSND effect, if the LOW solar solution [3] is accepted for c^2_{14}. Here, $\Delta m^2_{\rm LSND} = |\Delta m^2_{41}|$. Note, however, that the predicted value (14) of $\sin^2 2\theta_{\rm LSND}$ lies below its existing LSND lower limit 8×10^{-4} at 99% CL (when $s_{13} < 0.39$, what is consistent with $c^2_{13} \gg s^2_{13} \simeq 0$).

The assumptions on $(x_{ji})_{\rm sol}$, $(x_{ji})_{\rm atm}$ and $(x_{ji})_{\rm LSND}$ used above to derive Eqs. (7), (10) and (13) are all valid if *either*

$$m^2_1 \simeq m^2_2 \ll m^2_3 \ll m^2_4 \sim e.g.\;1\,{\rm eV}^2 \quad (15)$$

or

$$m^2_4 \ll m^2_1 \simeq m^2_2 \simeq m^2_3 \sim e.g.\;1\,{\rm eV}^2 \quad (16)$$

with

$$\Delta m^2_{21} \sim (10^{-5} \text{ or } 10^{-7} \text{ or } 10^{-10})\,\text{eV}^2 \quad , \quad \Delta m^2_{32} \sim 3 \times 10^{-3}\,\text{eV}^2 \tag{17}$$

in both cases. Then, in both cases the LSND mass-square scale is $\Delta m^2_{\rm LSND} = |\Delta m^2_{41}| \sim$ *e.g.* 1 eV^2 (of course, this value may be somewhat modified together with the values of m_1^2 and m_4^2).

Finally, for the Chooz reactor experiment [5], where it happens that $(x_{ji})_{\rm Chooz} \simeq (x_{ji})_{\rm atm}$, the first Eq. (6) predicts

$$\begin{aligned} P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\rm Chooz} &\simeq P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\rm atm} \\ &\simeq 1 - 2c^2_{13}s^2_{13}(1+c^2_{14})\sin^2(x_{32})_{\rm atm} - \frac{1}{2}c^2_{13}(1+s^2_{13}+c^2_{13}c^2_{14})s^2_{14}\,, \end{aligned} \tag{18}$$

since $\sin^2(x_{41})_{\rm atm} = 1/2$ and $\sin^2(x_{21})_{\rm atm} \ll \sin^2(x_{32})_{\rm atm}$ due to $|(x_{41})_{\rm atm}| \gg (x_{32})_{\rm atm} \sim 1 \gg (x_{21})_{\rm atm}$ with $|\Delta m^2_{41}| \gg \Delta m^2_{32} \gg \Delta m^2_{21}$. For $c^2_{13} \gg s^2_{13} \simeq 0$ the formula (18) is reduced to

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\rm Chooz} \simeq 1 - \frac{1}{2}(1+c^2_{14})s^2_{14}\,, \tag{19}$$

where $(1+c^2_{14})s^2_{14}/2 \sim 0.030$ if the LOW solar solution is used for c^2_{14}. In terms of the effective two-flavor oscillation formula, Eqs. (18) and (19) imply

$$\begin{aligned} \sin^2 2\theta_{\rm Chooz} \equiv \frac{P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\rm Chooz} - 1}{\sin^2 x_{\rm Chooz}} &= 4c^2_{13}s^2_{13}(1+c^2_{14})\sin^2(x_{32})_{\rm atm} + c^2_{13}(1+s^2_{13}+c^2_{13}c^2_{14})s^2_{14} \\ &\simeq (1+c^2_{14})s^2_{14}\,, \end{aligned} \tag{20}$$

the second step being valid for $c^2_{13} \gg s^2_{13} \simeq 0$. Here, $x_{\rm Chooz} = |(x_{41})_{\rm atm}| \gg 1$ and thus $\sin^2 x_{\rm Chooz} = 1/2$.

The negative result of Chooz experiment [5] excludes the disappearance process of reactor $\bar{\nu}_e$'s for $\sin^2 2\theta_{\rm Chooz} \stackrel{>}{\sim} 0.1$, when $\Delta m^2_{\rm Chooz} \stackrel{>}{\sim} 0.1\,\text{eV}^2$ is considered (then $\Delta m^2_{\rm Chooz} \gg \Delta m^2_{\rm atm} \sim 3 \times 10^{-3}\,\text{eV}^2$ and so, $x_{\rm Chooz} \gg x_{\rm atm} \sim 1$, leading consistently to $\sin^2 x_{\rm Chooz} = 1/2$). Thus, the nonobservation of Chooz effect in the above parameter range implies that $\sin^2 2\theta_{\rm Chooz} \stackrel{<}{\sim} 0.1$, when $\sin^2 x_{\rm Chooz} = 1/2$. Then, Eq. (20) requires

$$4c^2_{13}s^2_{13}(1+c^2_{14})\sin^2(x_{32})_{\rm atm} + c^2_{13}(1+s^2_{13}+c^2_{13}c^2_{14})s^2_{14} \stackrel{<}{\sim} 0.1 \tag{21}$$

or for $c_{13}^2 \gg s_{13}^2 \simeq 0$

$$(1 + c_{14}^2) s_{14}^2 \lesssim 0.1 \; . \tag{22}$$

So, in the case of Eq. (22) there must be

$$\frac{1}{2} s_{14}^4 \lesssim 1.3 \times 10^{-3} \; , \tag{23}$$

what due to the definition of $\sin^2 2\theta_{\rm LSND}$ in Eq. (14) gives the upper bound

$$\sin^2 2\theta_{\rm LSND} \simeq \frac{1}{2}(1 + 2s_{13}) s_{14}^4 \lesssim 1.3(1 + 2s_{13}) \times 10^{-3} \tag{24}$$

for the amplitude of LSND effect. Note that this Chooz-induced bound for the LSND effect allows its amplitude to be equal to the value (14) predicted by the use of LOW solar solution. However, as was already mentioned, such magnitude (14) of $\sin^2 2\theta_{\rm LSND}$ lies below its lower limit 8×10^{-4} following at 99% CL from the existing LSND data (when $s_{13} < 0.39$, what is consistent with $c_{13}^2 \gg s_{13}^2 \simeq 0$). Note also that the LMA and VAC solar solutions (in fact not applicable in our four-neutrino texture without changes), if used *on their face value*, lead to $\sin^2 2\theta_{\rm LSND} \sim 5.8(1+2s_{13}) \times 10^{-2}$ and $2.0(1+2s_{13}) \times 10^{-2}$, respectively. These figures are excluded by the Chooz-induced bound (24), though they are not inconsistent with the existing LSND data.

If $s_{13} \to 0$, our predictions (13) and (19) imply the amplitudes

$$\sin^2 2\theta_{\rm LSND} = \frac{1}{2} s_{14}^4 \quad , \quad \sin^2 2\theta_{\rm Chooz} = (1 + c_{14}^2) s_{14}^2 \tag{25}$$

which together with the amplitude $\sin^2 2\theta_{\rm sol} = c_{14}^2$ [as given in the zero perturbative order with respect to s_{14}^2/c_{14}^2 by Eq. (7)] lead to the sum rule

$$\sin^2 2\theta_{\rm sol} + \frac{1}{2} \sin^2 2\theta_{\rm Chooz} + \sin^2 2\theta_{\rm LSND} = 1 \tag{26}$$

for these three neutrino-oscillation amplitudes (each in the two-flavor approximation). This sum rule can be derived also from the probability summation relation $\sum_\beta P(\nu_e \to \nu_\beta)$ $= 1$ (with $\beta = e\,,\, \mu\,,\, \tau\,,\, s$) considered under the assumption of $m_1^2 \simeq m_2^2$ for solar ν_e's (when $|(x_{41})_{\rm sol}| \gg (x_{21})_{\rm sol} \simeq \pi/2$). The sum rule (26) leaves room for the LSND effect, depending on the magnitude of the Chooz effect which is not observed yet.

In conclusion, when accepting the present Chooz results, we face in the framework of our four-neutrino texture the alternative: *either* there is no LSND effect at all (then $s_{14} = 0$, and we are left with the popular three-neutrino nearly bimaximal texture working very well for solar ν_e's and atmospheric ν_μ's), *or* this effect exists but with an amplitude $\sin^2 2\theta_{\rm LSND} = (1+2s_{13})s_{14}^4/2$ reduced due to the Chooz-induced upper bound $1.3(1+2s_{13}) \times 10^{-3}$. If the two-flavor LOW solar solution is accepted for c_{14}^2, then $\sin^2 2\theta_{\rm LSND} \sim 4.5(1+2s_{13}) \times 10^{-4}$. The two-flavor LMA and VAC solar solutions cannot be applied in our four-neutrino texture without changes, because in our oscillation formula of solar ν_e's there appears a constant term, significant if these solutions are used. It spoils the familiar two-flavor structure of the oscillation formula. We hope that the Mini-BooNE experiment will be able to support or reject our simple four-neutrino texture, in particular its "sterile" perturbative aspect.

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